Hypothesis testing

Q1

It is known from previous data that 14% of the visitors to a particular cookery website are under 30 years of age.

To encourage more visitors under 30 years of age a large advertising campaign took place to target this age group.

To test whether the campaign was effective, a sample of 60 visitors to the website was selected. It was found that 15 of the visitors were under 30 years of age.

(a) Explain why a one-tailed hypothesis test should be used to decide whether the sample provides evidence that the campaign was effective.

(b) Carry out the hypothesis test at the 5% level of significance to investigate whether the sample provides evidence that the proportion of visitors under 30 years of age has increased.

Q2

It is known from historical data that 15% of the residents of a town buy the local weekly newspaper, 'Local News'.

A new free weekly paper is introduced into the town.

The owners of 'Local News' are interested to know whether the introduction of the free newspaper has changed the proportion of residents who buy their paper.

In a random sample of 50 residents of the town taken after the free newspaper was introduced, it was found that 3 of them purchased 'Local News' regularly.

Investigate, at the 5% significance level, whether this sample provides evidence that the proportion of local residents who buy 'Local News' has changed.

Q3

It is believed that a coin is biased so that the probability of obtaining a head when the coin is tossed is 0.7

(a) Assume that the probability of obtaining a head when the coin is tossed is indeed 0.7

Find the probability of obtaining exactly 6 heads from 7 tosses of the coin.

(b) Harry believes that the probability of obtaining a head for this coin is actually greater than 0.7

To test this belief he tosses the coin 35 times and obtains 28 heads.

Carry out a hypothesis test at the 10% significance level to investigate Harry's belief.

Q4

Naasir is playing a game with two friends. The game is designed to be a game of chance so that the probability of Naasir winning each game is $\frac{1}{3}$

Naasir claims he has a method to help him win more than $\frac{1}{3}$ of the games. To test this claim, the three of them played the game again 32 times and Naasir won 16 of these games.

(b) Stating your hypotheses clearly, test Naasir's claim at the 5% level of significance.

(4)

Q5

Afrika works in a call centre.

She assumes that calls are independent and knows, from past experience, that on each sales ca that she makes there is a probability of $\frac{1}{6}$ that it is successful.

Afrika makes 9 sales calls.

(a) Calculate the probability that at least 3 of these sales calls will be successful.

(2)

The probability of Afrika making a successful sales call is the same each day.

Afrika makes 9 sales calls on each of 5 different days.

(b) Calculate the probability that at least 3 of the sales calls will be successful on exactly 1 of these days.

(2)

Rowan works in the same call centre as Afrika and believes he is a more successful salesperson.

To check Rowan's belief, Afrika monitors the next 35 sales calls Rowan makes and finds that 11 of the sales calls are successful.

(c) Stating your hypotheses clearly test, at the 5% level of significance, whether or not there is evidence to support Rowan's belief.

(4)

Q6

A nursery has a sack containing a large number of coloured beads of which 14% are coloured red.

After several children have used beads from the sack, the nursery teacher decides to test whether or not the proportion of red beads in the sack has changed.
She takes a random sample of 75 beads and finds 4 red beads.

(*d*) Stating your hypotheses clearly, use a 5% significance level to carry out a suitable test for the teacher.

(4)

(*e*) Find the *p*-value in this case.

(1)

Q7

Past records show that 15% of customers at a shop buy chocolate. The shopkeeper believes that moving the chocolate closer to the till will increase the proportion of customers buying chocolate.

After moving the chocolate closer to the till, a random sample of 30 customers is taken and 8 of them are found to have bought chocolate.

Julie carries out a hypothesis test, at the 5% level of significance, to test the shopkeeper's belief.

Julie's hypothesis test is shown below.

$H_0 : p = 0.15$

$H_1 : p \geqslant 0.15$

Let X = the number of customers who buy chocolate.

$X \sim B(30, 0.15)$

$P(X = 8) = 0.0420$

$0.0420 < 0.05$ so reject H_0

There is sufficient evidence to suggest that the proportion of customers buying chocolate has increased.

(a) Identify the first two errors that Julie has made in her hypothesis test.

(2)

(b) Explain whether or not these errors will affect the conclusion of her hypothesis test. Give a reason for your answer.

(1)

(c) Find, using a 5% level of significance, the critical region for a one-tailed test of the shopkeeper's belief. The probability in the tail should be less than 0.05

(2)

(d) Find the actual level of significance of this test.

(1)

Q8

A manufacturer of sweets knows that 8% of the bags of sugar delivered from supplier A will be damp.
A random sample of 35 bags of sugar is taken from supplier A.

(a) Using a suitable model, find the probability that the number of bags of sugar that are damp is

(i) exactly 2

(ii) more than 3

(3)

Supplier B claims that when it supplies bags of sugar, the proportion of bags that are damp is less than 8%

The manufacturer takes a random sample of 70 bags of sugar from supplier B and finds that only 2 of the bags are damp.

(b) Carry out a suitable test to assess supplier B's claim.
You should state your hypotheses clearly and use a 10% level of significance.

(4)

Q9

Past information shows that 25% of adults in a large population have a particular allergy.

Rylan believes that the proportion that has the allergy differs from 25%

He takes a random sample of 50 adults from the population.

Rylan carries out a test of the null hypothesis H_0: $p = 0.25$ using a 5% level of significance.

(a) Write down the alternative hypothesis for Rylan's test.

(1)

(b) Find the critical region for this test.
You should state the probability associated with each tail, which should be as close to 2.5% as possible.

(4)

(c) State the actual probability of incorrectly rejecting H_0 for this test.

(1)

Rylan finds that 10 of the adults in his sample have the allergy.

(d) State the conclusion of Rylan's hypothesis test.

(1)

Q10

The random variable $X \sim B(27, 0.35)$

(a) Find (i) $P(X = 10)$

(ii) $P(12 \leqslant X < 15)$

(3)

Historical records show that the proportion of defective items produced by a machine is 0.12

Following a maintenance service of the machine, a random sample of 60 items is taken and 3 defective items are found.

(b) Carry out a suitable test to determine whether the proportion of defective items produced by the machine has decreased following the maintenance service.
You should state your hypotheses clearly and use a 5% level of significance.

(4)

(c) Write down the *p*-value for your test in part (b)

(1)
